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HEAT TRANSFER AND FRICTION FOR LAMINAR FLOW OF HELIUM AND CARBON DIOXIDE IN A CIRCULAR TUBE AT HIGH HEATING RATE*

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Abstract—Using an implicit finite-difference scheme described previously [1], solutions have been worked out for hydrodynamically developed laminar flow of helium and carbon dioxide with uniform heating rates so high that appreciable variations of gas properties occur. Whereas the results for helium closely approximate those previously found for air, both the Nusselt numbers and the friction factors for carbon dioxide come out considerably higher. For the Nusselt number with pure forced convection improved empirical expressions, based on the local Graetz number, are given. The local friction factors can be estimated from the wall-to-bulk temperature ratio.

NOMENCLATURE

c_p, specific heat at constant pressure;
D, tube diameter;
f, friction factor, $\tau_w/[\frac{1}{2}(\rho u)_m u_m]$;
g, acceleration of gravity;
k, thermal conductivity;
M, Mach number, based on mean axial velocity;
N_{Gr^*}, modified Grashof number, $g\rho^2 D^3/\mu^2$;
N_{Gz}, Graetz number, $(\pi/4)\, N_{Re} N_{Pr} D/x$;
$N_{Gz'}$, modified Graetz number, $N_{Re} N_{Pr} D/x$;
N_{Nu}, Nusselt number, $q''_w D/[k(T_w - T_m)]$;
N_{Pr}, Prandtl number, $\mu c_p/k$;
q''_w, heat flux at the wall;
q^+, non-dimensional heat flux, $q''_w D/(2k_o T_o)$;
T, absolute temperature;
u, axial velocity;
x, axial co-ordinate;
x^+, non-dimensional axial co-ordinate, $(2x/D)/N_{Re,o} N_{Pr,o})$;
μ, viscosity;
ρ, density;
τ_w, wall shear stress.

Subscripts
o, for gas properties, reference value at $x = 0$; for non-dimensional parameters, evaluated at $x = 0$;
m, mean value (with respect to cross-section); evaluated at the mixed mean temperature;
w, value at the wall;
cp, referring to flow with constant properties.

IN A PREVIOUS paper [1] a finite-difference method has been described for computing the laminar flow of gas in a circular tube with heating rates high enough to cause appreciable variations in the physical properties of the gas, and numerical examples—for flow with an unheated starting length—were given for air. The purpose of this note is to present solutions for gases with physical properties differing from those of air. Among the gases of interest in this connexion carbon dioxide and helium were chosen, the former because the variation with temperature

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of its transport properties differs markedly from what is found for air, the latter as representing the behaviour of the monatomic gases.

In the original computer program the temperature dependence of the specific heat, the viscosity, and the thermal conductivity was expressed as power laws,

$$c_p/c_{p,o} = (T/T_o)^a$$

$$\mu/\mu_o = (T/T_o)^b$$

$$k/k_o = (T/Y_o)^c$$

For helium the specific heat is independent of temperature, and the power laws, with $b = c = 0{\cdot}65$, represent quite accurately the variations of the transport coefficients as calculated using the Lennard-Jones potential and the method presented in Hirschfelder, Curtiss and Bird [2] and tabulated by McEligot [3]. In the case of carbon dioxide, however, the power-law approximations—in particular for the thermal conductivity—are not quite adequate. The computer program was, therefore, modified such that the viscosity and the thermal conductivity could be evaluated from more accurate expressions. For the viscosity Sutherland's law,

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{1 + S/T_o}{T/T_o + S/T_o}$$

was used. With $S = 230°K$ the calculated values agreed within 2 per cent with the values given by Hilsenrath *et al.* [4]. The thermal conductivity was expressed as

$$\frac{k}{k_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{1 + (1440/T_o)^2}{(T/T_o)^2 + (1440/T_o)^2}$$

This purely empirical expression represents the actual values between 200°K and 1400°K as given, in the low temperature range by Hilsenrath *et al.* [4], and at higher temperatures by Vines [5], with a maximum error of about 2 per cent.

The solutions were computed with inlet Mach numbers so small ($M_o \leqslant 0{\cdot}01$) that expansion work and viscous dissipation were negligible throughout the tube. The computations were carried out with $N_{Gr^*,o}/N_{Re,o} = 1$, corresponding to pure forced convection, and with $N_{Gr^*,o}/N_{Re,o} = 10^3$ which gives a significant free convection effect in the first part of the tube. For carbon dioxide both the limited data on the transport coefficients and the onset of dissociation restricts the maximum temperature for which the computations are valid to values below some 1500°K; consequently, only solutions for a moderate heat flux, $q^+ = 5$, were computed, whereas for helium compútions were carried out with $q^+ = 20$ also.

The results of the computations are summarized in Tables 1 and 2. From Fig. 1 which

Table 1. Wall parameters for helium

x^+	$N_{Nu,m}$	T_w/T_o	$f N_{Re,m}$	T_w/T_m	$N_{Gz,m}$
$q^+ = 5$;	$N_{Gr^*,o}/N_{Re,o} = 1$;		$M_o = 0{\cdot}5 \times 10^{-2}$		
0·001	17·10	1·597	25·26	1·566	1551
0·002	13·52	1·761	27·51	1·693	766
0·005	9·94	2·045	30·6	1·860	295
0·01	7·85	2·331	32·2	1·943	140
0·02	6·17	2·702	32·0	1·930	63·1
0·05	4·61	3·38	27·75	1·692	20·0
0·1	4·16	4·18	22·45	1·392	7·69
0·2	4·28	5·82	18·50	1·164	2·76
$q^+ = 5$;	$N_{Gr^*,o}/N_{Re,o} = 10^3$;		$M_o = 0{\cdot}65 \times 10^{-3}$		
0·001	18·21	1·562	32·2	1·532	1552
0·002	14·69	1·703	41·5	1·639	766
0·005	11·17	1·941	50·5	1·767	296
0·01	9·10	2·175	52·2	1·815	' 140
0·02	7·43	2·480	54·5	1·774	63·2
0·05	5·70	3·12	39·2	1·562	20·0
0·1	4·61	4·06	25·22	1·355	7·69
0·2	4·31	5·81	18·85	1·164	2·76
$q^+ = 20$;	$N_{Gr^*,o}/N_{Re,o} = 1$;		$M_o = 0{\cdot}5 \times 10^{-2}$		
0·001	18·61	3·12	51·0	2·889	1493
0·002	14·54	3·66	56·2	3·14	712
0·005	10·17	4·56	58·2	3·24	252
0·01	7·52	5·43	53·2	3·00	107
0·02	5·50	6·51	43·2	2·491	42·1
0·05	4·09	8·45	28·19	1·682	11·0
0·1	4·20	11·30	20·01	1·253	3·76

Table 1—continued

$q^+ = 20$;	$N_{Gr^*,o}/N_{Re,o} = 10^3$;		$M_o = 0.65 \times 10^{-3}$		
x^+	$N_{Nu,m}$	T_w/T_o	$fN_{Re,m}$	T_w/T_m	$N_{Gz,m}$
0·001	20·22	2·962	70·2	2·748	1496
0·002	16·00	3·43	79·4	2·961	714
0·005	11·38	4·23	81·7	3·02	253
0·01	8·47	5·02	70·9	2·792	107
0·02	6·16	6·09	52·2	2·342	42·2
0·05	4·37	8·22	29·43	1·642	11·0
0·1	4·23	11·27	20·13	1·252	3·76

Table 2. Wall parameters for carbon dioxide

$q^+ = 5$;	$N_{Gr^*,o}/N_{Re,o} = 1$;		$M_o = 10^{-2}$		
x^+	$N_{Nu,m}$	T_w/T_o	$fN_{Re,m}$	T_w/T_m	$N_{Gz,m}$
0·001	19·36	1·522	25·91	1·493	1536
0·002	15·54	1·649	28·00	1·586	751
0·005	11·50	1·859	30·4	1·692	282
0·01	8·96	2·062	31·1	1·726	128
0·02	6·79	2·317	29·84	1·677	54·7
0·05	4·77	2·774	24·76	1·457	15·6
0·1	4·32	3·33	20·10	1·236	5·70
0·2	4·36	4·54	17·58	1·104	2·17
$q^+ = 5$;	$N_{Gr^*,o}/N_{Re,o} = 10^3$;		$M_o = 10^{-3}$		
x^+	$N_{Nu,m}$	T_w/T_o	$fN_{Re,m}$	T_w/T_m	$N_{Gz,m}$
0·001	20·47	1·495	35·7	1·467	1537
0·002	16·70	1·606	41·6	1·546	752
0·005	12·72	1·786	49·3	1·627	282
0·01	10·19	1·957	52·5	1·640	128
0·02	8·01	2·174	49·9	1·576	54·8
0·05	5·76	2·623	33·9	1·380	15·6
0·1	4·64	3·28	22·21	1·221	5·71
0·2	4·37	4·54	17·87	1·104	2·17

shows the local Nusselt numbers together with some of the results previously obtained for air, one sees that the different forms of the temperature dependence of the transport properties do indeed give rise to differences in heat transfer. In particular the results for carbon dioxide deviate significantly from what is found for air and helium; while the differences between the latter two are of the order of 3–5 per cent, carbon dioxide gives up to 18 per cent higher values in the entrance region. One also notes that the increased heat transfer to carbon dioxide causes a faster approach to the asymptotic value.

McEligot and Swearingen [6] have proposed a relatively simple empirical equation for the heat transfer with uniform heat flux and pure forced convection, based on the numerical solutions for air presented in reference [1],

$$\frac{N_{Nu,m}}{N_{Nu,cp}} = [1 - 0{\cdot}00125q^{+3/2}]\,(N_{Gz',m})^{0\cdot0025q^+}$$

Using this expression in the range $1000 > N_{Gz',m} > 60$ and the solution for constant properties for $N_{Gz',m} < 60$, one obtains values for the Nusselt number that agree within some 7 per cent with the numerical results.

The introduction of the local Graetz number, based on properties at the mean gas temperature, constitutes a significant improvement over the use of the purely geometrical quantity x^+ in reference [1]. However, if an additive, rather than a multiplicative, correction is made to the constant-properties Nusselt numbers, more accurate expressions may be obtained. The following equations thus represent the data for pure forced convection within some 3 per cent.

Air and Helium:

$$N_{Nu,m} = N_{Nu,cp} + 0{\cdot}025q^{+1/2}(N_{Gz,m} - 3) \times (N_{Gz,m} - 20)/N^{3/2}_{Gz,m}$$

for $1000 > N_{Gz,m} > 3$ and $0 \leqslant q^+ \leqslant 20$;

$$N_{Nu,m} = N_{Nu,cp} \quad \text{for} \quad N_{Gz,m} \leqslant 3.$$

Carbon dioxide:

$$N_{Nu,m} = N_{Nu,cp} + 0{\cdot}07q^{+1/2}(N_{Gz,m} - 8)/N^{1/2}_{Gz,m}$$

for $1000 > N_{Gz,m} > 10$ and $0 \leqslant q^+ \leqslant 5$;

$$N_{Nu,m} = N_{Nu,cp} \quad \text{for} \quad N_{Gz,m} \leqslant 10.$$

For superimposed free and forced convection the Nusselt number depends in a complex way on N_{Gz}, N_{Gr^*}/N_{Re}, and q^+. For very small values of q^+ the natural convection effect of

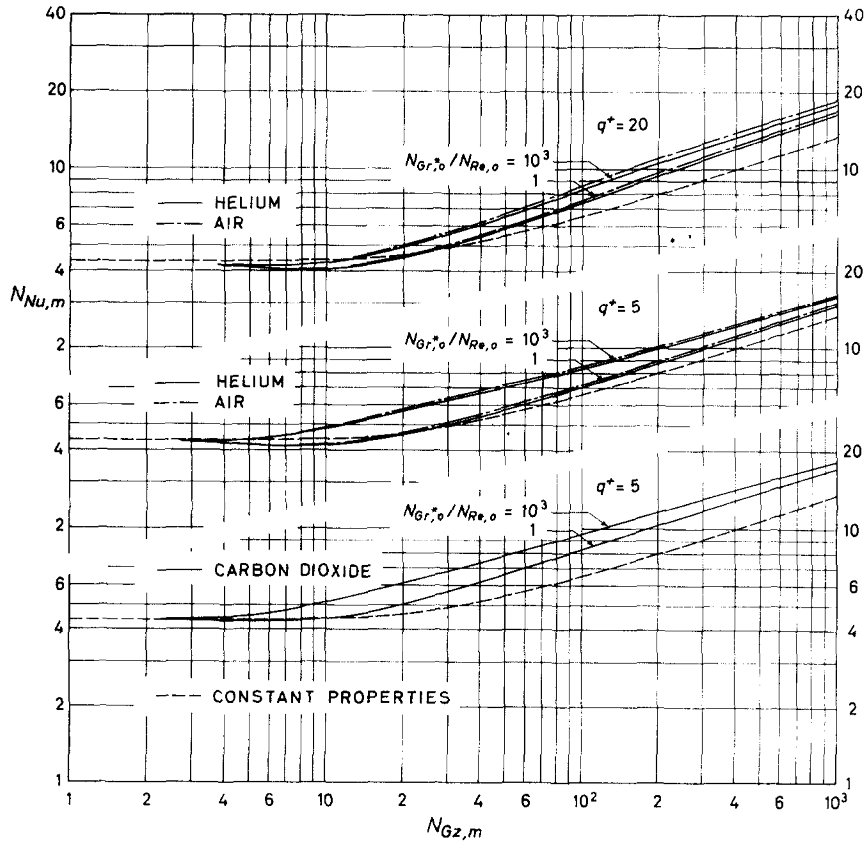

FIG. 1. Local Nusselt numbers for helium, carbon dioxide, and air together with the solution for constant properties [7].

course vanishes; what is perhaps more surprising is the fact that the effect also diminishes at very high heat fluxes as may be seen from Fig. 1. The explanation is that the local value of N_{Gr^*}/N_{Re}, evaluated at the mean gas temperature, decreases rapidly with increasing temperature.

For moderate wall-to-bulk temperature ratios the friction factors for both helium and carbon dioxide agree well with the expression given in reference [1] for air,

$$fN_{Re,m} = 16(T_w/T_b).$$

At higher values of the temperature ratio the friction factor for carbon dioxide shows a somewhat stronger increase than is the case for air and helium. Whereas for the latter the agreement with the expression given in reference [1],

$$fN_{Re,m} = 15{\cdot}5(T_w/T_b)^{1{\cdot}10}$$
$$\text{for} \quad 1{\cdot}5 < T_w/T_b < 3,$$

is excellent, the data for carbon dioxide requires a larger exponent,

$$fN_{Re,m} = 15{\cdot}5(T_w/T_b)^{1{\cdot}25}$$
$$\text{for} \quad 1{\cdot}2 < T_w/T_b < 2.$$

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Résumé—On a obtenu, à l'aide d'une méthode par différence finies décrite auparavant [1], des solutions pour l'écoulement laminaire établi hydrodynamiquement d'hélium et de gaz carbonique avec des flux de chauffage uniforme si élevés qu'il se produit des variations notables des propriétés des gaz. Tandis que les résultats pour l'hélium sont très voisins de ceux obtenus auparavant pour l'air, les nombres de Nusselt et les coefficients de frottement pour le gaz carbonique sont considérablement plus élevés. Des expressions empiriques améliorées, basées sur le nombre local de Graetz, sont données pour le nombre de Nusselt avec convection forcée seule. Les coefficients de frottement locaux peuvent être estimés à partir du rapport entre la température pariétale et la température moyenne.

Zusammenfassung—Unter Benutzung eines früher veröffentlichten Differenzenverfahrens [1] wurden Lösungen für hydrodynamisch ausgebildete Strömungen von Helium und Karbondioxyd bei konstanter Wärmestromdichte unter Berücksichtigung temperaturabhängiger Stoffwerte erhalten. Während die Ergebnisse für Helium sehr gut mit den früheren für Luft übereinstimmen, wurden für die Nusselt-Zahlen und die Reibungskoeffizienten für Karbondioxyd beträchtlich höhere Werte gefunden. Für die Nusselt-Zahlen bei reiner Konvektion wurden verbesserte empirische Ausdrücke beruhend auf die örtlichen Graetz-Zahlen angegeben. Die örtlichen Reibungskoeffizienten können aus dem Verhältnis der Wandtemperatur zur Mischtemperatur berechnet werden.

Аннотация—С помощью неявной конечно-разностной схемы описанной, ранее [1], получены решения для гидродинамически развитого ламинарного потока гелия и углекислого газа с такой большой постоянной скоростью нагрева, что происходят заметные изменения свойств газа. Тогда как результаты для гелия близки к результатам, ранее полученным для воздуха, числа Нуссельта и коэффициенты трения для углекислого газа получаются значительно выше. Для числа Нуссельта при сичто вынужденной конвекции приводятся уточненные эмпирические выражения, включающие локальное число Грэтца. Локальный коэффициент трения может быть оценен через перепад температур между стенкой и потоком.